1-23. Use the ε_{ijk} notation and derive the identity

$$(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{ABD})\mathbf{C} - (\mathbf{ABC})\mathbf{D}$$

1-24. Let **A** be an arbitrary vector, and let **e** be a unit vector in some fixed direction. Show that

$$\mathbf{A} = \mathbf{e}(\mathbf{A} \cdot \mathbf{e}) + \mathbf{e} \times (\mathbf{A} \times \mathbf{e})$$

What is the geometrical significance of each of the two terms of the expansion?

1-25. Find the components of the acceleration vector **a** in spherical coordinates.

1-26. A particle moves with $v =$ const. along the curve $r = k(1 + \cos\theta)$ (a *cardioid*). Find $\ddot{\mathbf{r}} \cdot \mathbf{e}_r = \mathbf{a} \cdot \mathbf{e}_r, |\mathbf{a}|$, and $\dot{\theta}$.

1-27. If **r** and $\dot{\mathbf{r}} = \mathbf{v}$ are both explicit functions of time, show that

$$\frac{d}{dt}[\mathbf{r} \times (\mathbf{v} \times \mathbf{r})] = r^2\mathbf{a} + (\mathbf{r} \cdot \mathbf{v})\mathbf{v} - (v^2 + \mathbf{r} \cdot \mathbf{a})\mathbf{r}$$

1-29. Find the angle between the surfaces defined by $r^2 = 9$ and $x + y + z^2 = 1$ at the point $(2, -2, 1)$.

1-34. Evaluate the integral

$$\int \mathbf{A} \times \ddot{\mathbf{A}} dt$$

1-37. Find the value of the integral $\int_S \mathbf{A} \cdot d\mathbf{a}$, where $\mathbf{A} = (x^2 + y^2 + z^2)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$ and the surface S is defined by the sphere $R^2 = x^2 + y^2 + z^2$. Do the integral directly and also by using Gauss's theorem.

1-40. The height of a hill in meters is given by $z = 2xy - 3x^2 - 4y^2 - 18x + 28y + 12$, where x is the distance east and y is the distance north of the origin. **(a)** Where is the top of the hill and how high is it? **(b)** How steep is the hill at $x = y = 1$, that is, what is the angle between a vector perpendicular to the hill and the z axis? **(c)** In which compass direction is the slope at $x = y = 1$ steepest?

2-2. A particle of mass m is constrained to move on the surface of a sphere of radius R by an applied force $\mathbf{F}(\theta, \phi)$. Write the equation of motion.

1.41 An astronaut in gravity-free space is twirling a mass m on the end of a string of length R in a circle, with constant angular velocity ω. Write down Newton's second law in polar coordinates and find the tension in the string.

a. What is the tension in the string if the astronaut lets it out at a constant rate? If the astronaut allows the string to double in length, what happens to $\dot{\phi}$? (Use $\vec{F} = m\vec{a}$ in polar coordinates. Hint: what is F_ϕ?)
b. Suppose the astronaut lets go of the string. What is r(t)? (Use geometry to figure this out.) Show that your geometric solution does, in fact, satisfy $\vec{F} = m\vec{a}$ in polar coordinates.